

## Joint posterior density

The joint posterior density of all unknowns under DFMH can be written as follows:

$$p(\boldsymbol{\beta}, g_1, g_2, \dots, g_m, \sigma_{g_1}^2, \sigma_{g_1}^2, \dots, \sigma_{g_m}^2, \sigma_e^2, s^2, \nu, \pi | \mathbf{y}) \propto \left( \prod_{i=1}^n p(y_i | \boldsymbol{\beta}, g_1, g_2, \dots, g_m, \sigma_e^2) \right) \left( \prod_{j=1}^m p(g_j | \sigma_{g_j}^2) p(\sigma_{g_j}^2 | \nu, s^2, \pi) \right) p(s_g^2 | \alpha_s, \beta_s) p(\nu_g) p(\pi | \alpha_\pi, \beta_\pi) p(\boldsymbol{\beta}) p(\sigma_e^2)$$

where

$$p(y_i | \boldsymbol{\beta}, g_1, g_2, \dots, g_m, \sigma_e^2) = (2\pi\sigma_e^2)^{-n/2} \exp\left(-\frac{1}{2\sigma_e^2} \sum_{i=1}^n \left(y_i - \mathbf{x}_i' \boldsymbol{\beta} - \sum_{j=1}^m z_{ij} g_j\right)^2\right),$$

$$p(g_j | \sigma_{g_j}^2) = (2\pi\sigma_{g_j}^2)^{-1/2} \exp\left(-\frac{1}{2\sigma_{g_j}^2} g_j^2\right),$$

$$p(\sigma_{g_j}^2 | \nu, s^2, \pi) = \begin{cases} \frac{\left(\frac{\nu s^2}{2}\right)^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)} \sigma_{g_i}^2^{-\left(\frac{\nu}{2}+1\right)} e^{-\frac{\nu s^2}{2\sigma_{g_i}^2}} & \text{with probability } \pi, \\ 0 & \text{with probability } (1 - \pi) \end{cases}$$

$$p(s_g^2 | \alpha_s, \beta_s) = \frac{(\beta_s)^{\alpha_s}}{\Gamma(\alpha_s)} (s^2)^{\alpha_s-1} e^{-\beta_s s^2},$$

$$p(\nu_g) = \frac{1}{(1 + \nu_g)^2},$$

$$p(\pi | \alpha_\pi, \beta_\pi) \propto \pi^{\alpha_\pi - 1} (1 - \pi)^{\beta_\pi - 1},$$

and  $p(\boldsymbol{\beta})$  and  $p(\sigma_e^2)$  can be arbitrarily specified to be informative or diffuse priors with different functional distributional forms.

For UNIMH and BIVMH, the joint posterior density is further marginalized to integrate out the uncertainty on  $\sigma_{g_1}^2, \sigma_{g_2}^2, \dots, \sigma_{g_m}^2$ ; i.e.

$$p(\boldsymbol{\beta}, g_1, g_2, \dots, g_m, \sigma_e^2, s^2, \nu, \pi | \mathbf{y}) \propto \left( \prod_{i=1}^n p(y_i | \boldsymbol{\beta}, g_1, g_2, \dots, g_m, \sigma_e^2) \right) \left( \prod_{j=1}^m p(g_j | \nu, s^2, \pi) \right) p(s_g^2 | \alpha_s, \beta_s) p(\nu_g) p(\pi | \alpha_\pi, \beta_\pi) p(\boldsymbol{\beta}) p(\sigma_e^2)$$

where

$$p(g_j | \nu, s^2, \pi) = \int_{\sigma_{g_j}^2} p(g_j | \sigma_{g_j}^2) p(\sigma_{g_j}^2 | \nu, s^2, \pi) d\sigma_{g_j}^2$$

$$= \begin{cases} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left(\frac{1}{\pi \nu s^2}\right)^{1/2} \left[1 + \frac{g_j^2}{\nu s^2}\right]^{-\frac{\nu+1}{2}} & \text{with probability } \pi \\ 0 & \text{with probability } (1-\pi) \end{cases}$$

i.e., a mixture of a scaled Student  $t$  distribution with probability  $\pi$  and a point mass at zero of probability  $(1-\pi)$ .

## Description of the three sampling strategies

### *Sampling strategy for DFMH*

**Sampling the degrees of freedom parameter for the random SNP effects:** We used a proper prior  $p(\nu) \propto (\nu+1)^{-2}$  which corresponds to a Uniform(0,1) prior on  $(\nu+1)^{-1}$ . The full conditional density (FCD) for  $\nu$  can be written as follows:

$$p(\nu | ELSE) \propto \left( \prod_{j=1}^m I(\sigma_{g_j}^2 > 0) p(\sigma_{g_j}^2 | \nu, s^2) \right) p(\nu)$$

$$= \left( \prod_{j=1}^m I(\sigma_{g_j}^2 > 0) \frac{\left( \frac{\nu s^2}{2} \right)^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)} \sigma_{g_j}^{2 - \left(\frac{\nu}{2} + 1\right)} e^{-\frac{\nu s^2}{2\sigma_{g_j}^2}} \right) \frac{1}{(1+\nu)^2}$$

where  $I(.)$  is the indicator variable such that if the condition inside  $(.)$  is true,  $I(.) = 1$ , otherwise  $I(.) = 0$ . As this FCD is not recognizable, we propose a random walk normal Metropolis Hastings (MH) step on  $\xi = \log(\nu)$ . Noting that the Jacobian from  $\nu$  to  $\xi$  is  $\exp(\xi)$ , the corresponding FCD for  $\xi$  is as follows:

$$p(\xi | ELSE)$$

$$\propto \left( \frac{\left( \exp(\xi) s^2 / 2 \right)^{\exp(\xi)/2}}{\Gamma(\exp(\xi) / 2)} \right)^{m_1} \left( \prod_{j=1}^m I(\sigma_{g_j}^2 > 0) \sigma_{g_j}^{2 - \left(\frac{\exp(\xi)}{2} + 1\right)} e^{-\frac{\exp(\xi) s^2}{2\sigma_{g_j}^2}} \right) \frac{1}{(1 + \exp(\xi))^2} \exp(\xi)$$

where  $m_1 = \sum_{j=1}^m I(\sigma_{g_j}^2 > 0)$ . Hence

$$\begin{aligned}
& \log p(\xi | ELSE) \\
&= m_1 \left( \frac{\exp(\xi)}{2} (\xi + \log(s^2) - \log(2)) - \log \Gamma \left( \frac{\exp(\xi)}{2} \right) \right) + \\
& \sum_{j=1}^m I(\sigma_{g_j}^2 > 0) \left( - \left( \frac{\exp(\xi)}{2} + 1 \right) \log(\sigma_{g_j}^2) - \frac{\exp(\xi) s^2}{2 \sigma_{g_j}^2} \right) - 2 \log(1 + \exp(\xi)) + \xi
\end{aligned}$$

Suppose the value of  $\xi$  in the current cycle  $i$  is  $\xi^{[i]}$ , the random walk proposal for  $\xi^{[i+1]}$  in the next MCMC cycle is drawn from a Gaussian distribution:

$$p(\xi^*) = \frac{1}{\sqrt{2\pi}c_v} \exp \left( - \frac{(\xi^* - \xi^{[i]})^2}{2c_v^2} \right)$$

This is equivalent to generating a random variable, say  $\delta$  from  $N(0, c_v^2)$  and adding it to  $\xi^{[i]}$  to

propose  $\xi^* = \xi^{[i]} + \delta$ . The MH acceptance ratio is determined to be  $\alpha = \frac{p(\xi^* | ELSE)}{p(\xi^{[i]} | ELSE)}$ . For

numerical stability, we evaluated this ratio as:

$$\alpha = \exp \left( \log p(\xi^* | ELSE) - \log p(\xi^{[i]} | ELSE) \right)$$

To implement this MH sampling strategy, we generate  $u$  from a Uniform(0,1) distribution. If

$\alpha > 1$ , accept  $\xi^{[i+1]} = \xi^*$ . If  $u < \alpha$  then set  $\xi^{[i+1]} = \xi^*$ . Otherwise if  $u > \alpha$ , then set  $\xi^{[i+1]} = \xi^{[i]}$ .

The following tuning procedure adapted from Muller [1] is to determine  $c_v^2$ :

- 1) For every 10 cycles, if the rate of acceptance is greater than 80%, increase  $c_v^2$  by a factor of 1.2.

2) For every 10 cycles if the rate of acceptance is less than 20%, decrease  $c_v^2$  by a factor of 0.7.

3) After burn-in, keep  $c_v^2$  constant and monitor subsequent acceptance rates to ensure that they fall within 25 to 75%.

**Sampling the scale parameter for the random SNP effects:** Borrowing results from Yi and Xu

[2], the FCD for  $s^2$  based on the specification of a conjugate prior  $p(s^2 | \alpha_s, \beta_s) = \text{Gamma}$

$(\alpha_s, \beta_s)$  can be written as follows:

$$\begin{aligned}
 p(s^2 | ELSE) &\propto \left( \prod_{j=1}^m I(\sigma_{g_j}^2 > 0) p(\sigma_{g_j}^2 | \nu, s^2) \right) p(s^2 | \alpha_s, \beta_s) \\
 &= \left( \prod_{j=1}^m I(\sigma_{g_j}^2 > 0) \frac{\left(\frac{s^2 \nu}{2}\right)^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)} \sigma_{g_j}^2^{-\left(\frac{\nu}{2}+1\right)} e^{-\frac{s^2 \nu}{2\sigma_{g_j}^2}} \right) \frac{(\beta_s)^{\alpha_s}}{\Gamma(\alpha_s)} (s^2)^{\alpha_s-1} e^{-\beta_s s^2} \\
 &\propto (s^2)^{\alpha_s + \frac{m_1 \nu}{2} - 1} \exp\left(-s^2 \left(\frac{\nu}{2} \sum_{j=1}^m I(\sigma_{g_j}^2 > 0) \sigma_{g_j}^{-2} + \beta_s\right)\right)
 \end{aligned}$$

i.e., a Gamma distribution with parameters  $\alpha_s + \frac{m_1 \nu}{2}$  and  $\frac{\nu}{2} \sum_{j=1}^m I(\sigma_{g_j}^2 > 0) \sigma_{g_j}^{-2} + \beta_s$ .

### *Sampling strategy for UNIMH*

**Sampling the degrees of freedom parameter for the random SNP effects:** The “collapsed”

FCD for sampling  $\nu$  (after integrating out  $\{\sigma_{g_j}^2\}_{j=1}^m$  from the FCD for  $\nu$  as specified in DFMH

above) is as follows:

$$p(\nu | ELSE) \propto \left( \prod_{j=1}^m I(g_j \neq 0) p(g_j | \nu, s_g^2) \right) p(\nu)$$

$$= \left[ \prod_{j=1}^m I(g_j \neq 0) \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left(\frac{1}{\pi \nu s^2}\right)^{1/2} \left[1 + \frac{g_j^2}{\nu s^2}\right]^{-\frac{\nu+1}{2}} \right] \frac{1}{(1+\nu)^2}$$

As this FCD is not recognizable, we specify a random walk MH step on  $\xi = \log(\nu)$ . Note that the Jacobian from  $\nu$  to  $\xi$  is  $\exp(\xi)$ . The corresponding FCD for  $\xi$  is as follows:

$$p(\xi | ELSE)$$

$$\propto \left( \frac{\Gamma((\exp(\xi)+1)/2)}{\Gamma(\exp(\xi)/2)} \right)^{m_1} \left( \frac{1}{\pi \exp(\xi) s^2} \right)^{m_1/2} \left( \prod_{j=1}^m I(g_j \neq 0) \left[ 1 + \frac{g_j^2}{\exp(\xi) s^2} \right]^{-\frac{\exp(\xi)+1}{2}} \right)$$

$$\frac{1}{(1+\exp(\xi))^2} \exp(\xi)$$

where  $m_1 = \sum_{j=1}^m I(g_j \neq 0)$ . Hence

$$\begin{aligned}
& \log p(\xi | ELSE) \\
&= m_1 \left( \log \Gamma\left(\frac{\exp(\xi)+1}{2}\right) - \log \Gamma\left(\frac{\exp(\xi)}{2}\right) + \frac{1}{2} \log\left(\frac{1}{\pi \exp(\xi) s^2}\right) \right) + \\
& \sum_{j=1}^m I(g_j \neq 0) \left( -\left(\frac{\exp(\xi)+1}{2}\right) \log\left(1 + \frac{g_j^2}{\exp(\xi) s^2}\right) \right) - 2 \log(1 + \exp(\xi)) + \xi
\end{aligned}$$

Suppose the value of  $\xi$  in the current cycle  $i$  is  $\xi^{[i]}$ . We propose a random walk sample for

$\xi^{[i+1]}$  in the next cycle using a Gaussian proposal distribution:

$$p(\xi^*) = \frac{1}{\sqrt{2\pi}c_v} \exp\left(\frac{-\left(\xi^* - \xi^{[i]}\right)^2}{2c_v^2}\right)$$

That is equivalent to generating a random variable, say  $\delta$  from  $N(0, c_v^2)$  and adding it to  $\xi^{[i]}$  to

propose  $\xi^* = \xi^{[i]} + \delta$ . To determine the MH acceptance ratio  $\alpha = \frac{p(\xi^* | ELSE)}{p(\xi^{[i]} | ELSE)}$ , we evaluated

this ratio in a numerically stable manner as:

$$\alpha = \exp\left(\log p(\xi^* | ELSE) - \log p(\xi^{[i]} | ELSE)\right)$$

To implement this MH sampling strategy, we first generated  $u$  from a Uniform(0,1) distribution.

If  $\alpha > 1$  or  $u < \alpha$  then set  $\xi^{[i+1]} = \xi^*$  otherwise set  $\xi^{[i+1]} = \xi^{[i]}$ . The same tuning procedure from

Muller [1] as described in DFMH for  $c_v^2$  was used here.

**Sampling the scale parameter for the random SNP effects:** The FCD for sampling  $s^2$  is as follows:

$$\begin{aligned}
p(s^2 | ELSE) &\propto \left( \prod_{j=1}^m I(g_j \neq 0) p(g_j | \nu, s^2) \right) p(s^2 | \alpha_s, \beta_s) \\
&= \left( \prod_{j=1}^m I(g_j \neq 0) \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left(\frac{1}{\pi \nu s^2}\right)^{1/2} \left[1 + \frac{g_j^2}{\nu s^2}\right]^{-\frac{\nu+1}{2}} \right) \frac{(\beta_s)^{\alpha_s}}{\Gamma(\alpha_s)} (s^2)^{\alpha_s-1} e^{-\beta_s s^2}
\end{aligned}$$

Even though this FCD is recognizable, we propose a random walk MH step on  $\psi = \log(s^2)$ . Note that the Jacobian from  $s^2$  to  $\psi$  is  $\exp(\psi)$ . The corresponding FCD for  $\psi$  is as follows:

$$\begin{aligned}
p(\psi | ELSE) &= \left( \prod_{j=1}^m I(g_j \neq 0) \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left(\frac{1}{\pi \nu s^2}\right)^{1/2} \left[1 + \frac{g_j^2}{\nu s^2}\right]^{-\frac{\nu+1}{2}} \right) \frac{(\beta_s)^{\alpha_s}}{\Gamma(\alpha_s)} (s^2)^{\alpha_s-1} e^{-\beta_s s^2} \\
&\propto \left( \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)} \right)^{m_1} \left( \frac{1}{\pi \nu \exp(\psi)} \right)^{m_1/2} \left( \prod_{j=1}^m I(g_j \neq 0) \left[1 + \frac{g_j^2}{\nu \exp(\psi)}\right]^{-\frac{\nu+1}{2}} \right) \\
&(\exp(\psi))^{\alpha_s-1} e^{-\beta_s \exp(\psi)} \exp(\psi)
\end{aligned}$$

where  $m_1 = \sum_{j=1}^m I(g_j \neq 0)$ . Hence

$$\begin{aligned}
&\log p(\psi | ELSE) \\
&= m_1 \left( \log \Gamma\left(\frac{\nu+1}{2}\right) - \log \Gamma\left(\frac{\nu}{2}\right) + \frac{1}{2} \log\left(\frac{1}{\pi \nu \exp(\psi)}\right) \right) + \\
&\sum_{j=1}^m I(g_j \neq 0) \left( -\left(\frac{\nu+1}{2}\right) \log\left(1 + \frac{g_j^2}{\nu \exp(\psi)}\right) \right) + (\alpha_s - 1)\psi - \beta_s \exp(\psi) + \psi
\end{aligned}$$

Suppose the value of  $\psi$  in the current cycle  $i$  is  $\psi^{[i]}$ . We propose a random walk value for

$\psi^{[i+1]}$  in the next cycle from a Gaussian distribution:



$$p(\psi^*) = \frac{1}{\sqrt{2\pi}c_s} \exp\left(\frac{-(\psi^* - \psi^{[i]})^2}{2c_s^2}\right)$$

That is equivalent to generate a random variable, say  $\delta$  from  $N(0, c_s^2)$  and adding it to  $\psi^{[i]}$  to

propose  $\psi^* = \psi^{[i]} + \delta$ . To determine the acceptance ratio  $\alpha = \frac{p(\psi^* | ELSE)}{p(\psi^{[i]} | ELSE)}$  in a numerically

stable manner, we evaluated this ratio as:

$$\alpha = \exp\left(\log p(\psi^* | ELSE) - \log p(\psi^{[i]} | ELSE)\right)$$

To implement this MH sampling strategy, we first generated  $u$  from a Uniform(0,1) distribution.

If  $\alpha > 1$  or  $u < \alpha$ , set  $\psi^{[i+1]} = \psi^*$ . Otherwise, if  $u > \alpha$  then set  $\psi^{[i+1]} = \psi^{[i]}$ . The same tuning

procedure from Muller [1] described previously is used to tune  $c_s^2$ :

### *Sampling strategy for BIVMH*

#### **Sampling the degrees of freedom and scale parameters for the random SNP effects:**

We divided burn-in into four stages with equal length:

**Stage 1:** Sample  $\log(\nu)$  and  $\log(s^2)$  using UNIMH (see sampling strategy 2) with fine-tuning procedure on  $c_\nu^2$  and  $c_s^2$ , which are also the variances for the two separate Gaussian proposal densities;

**Stage 2:** Sample  $\log(\nu)$  and  $\log(s^2)$  using UNIMH with fixing  $c_\nu^2$  and  $c_s^2$  to the values tuned from the last cycle in stage 1 and compute correlation  $r$  between samples of  $\log(\nu)$  and  $\log(s^2)$  within stage 2;

**Stage 3:** Jointly sample  $\log(\nu)$  and  $\log(s^2)$  using a bivariate Gaussian proposal density with variances  $c_\nu^2$  and  $c_s^2$  based on those tuned at the end of Stage 1 and a covariance based on the correlation computed from Stage 2. The joint FCD for  $\nu$  and  $s^2$  (based on a “collapsed specification that integrates out  $\{\sigma_{g_j}^2\}_{j=1}^m$  from the FCD) is as follows:

$$p(\nu, s^2 | ELSE) \propto \left( \prod_{j=1}^m I(g_j \neq 0) p(g_j | \nu, s^2) \right) p(\nu) p(s^2 | \alpha_s, \beta_s)$$

$$= \left( \prod_{j=1}^m I(g_j \neq 0) \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left(\frac{1}{\pi \nu s^2}\right)^{1/2} \left[1 + \frac{g_j^2}{\nu s^2}\right]^{-\frac{\nu+1}{2}} \right) \frac{1}{(1+\nu)^2} \frac{(\beta_s)^{\alpha_s}}{\Gamma(\alpha_s)} (s^2)^{\alpha_s-1} e^{-\beta_s s^2}$$

As this density is not recognizable, we could use a random walk normal MH step on  $\xi = \log(\nu)$  and  $\psi = \log(s^2)$ . Note that the Jacobian from  $\nu$  to  $\xi$  is  $\exp(\xi)$  whereas the Jacobian from  $s^2$  to  $\psi$  is  $\exp(\psi)$ . The corresponding FCD for  $\xi$  and  $\psi$  is as follows:

$$p(\xi, \psi | ELSE)$$

$$= \left( \frac{\Gamma((\exp(\xi)+1)/2)}{\Gamma(\exp(\xi)/2)} \right)^{m_1} \left( \frac{1}{\pi \exp(\xi) \exp(\psi)} \right)^{m_1/2} \left( \prod_{j=1}^m I(g_j \neq 0) \left[ 1 + \frac{g_j^2}{\exp(\xi) \exp(\psi)} \right]^{-\frac{\exp(\xi)+1}{2}} \right)$$

$$\frac{1}{(1+\exp(\xi))^2} (\exp(\psi))^{\alpha_s-1} e^{-\beta_s \exp(\psi)} \exp(\psi) \exp(\xi)$$

Where  $m_1 = \sum_{j=1}^m I(g_j \neq 0)$ , hence

$$\begin{aligned} & \log p(\xi, \psi | ELSE) \\ &= m_1 \left( \log \Gamma \left( \frac{\exp(\xi) + 1}{2} \right) - \log \Gamma \left( \frac{\exp(\xi)}{2} \right) + \frac{1}{2} \left( \frac{1}{\pi \exp(\xi) \exp(\psi)} \right) \right) + \\ & \sum_{j=1}^m I(g_j \neq 0) \left( - \left( \frac{\exp(\xi) + 1}{2} \right) \log \left( 1 + \frac{g_j^2}{\exp(\xi) \exp(\psi)} \right) \right) - \\ & 2 \log(1 + \exp(\xi)) + (\alpha_s - 1) \psi - \beta_s \exp(\psi) + \psi + \xi \end{aligned}$$

Suppose the value of  $\boldsymbol{\eta} = [\xi, \psi]'$  in the current cycle  $i$  is  $\boldsymbol{\eta}^{[i]}$ . We propose a random walk for  $\boldsymbol{\eta}^{[i+1]}$  in the next cycle from a bivariate Gaussian distribution:

$$p(\boldsymbol{\eta}^*) = \frac{1}{2\pi |c_{\boldsymbol{\eta}}^2 \boldsymbol{\Sigma}|^{1/2}} \exp \left( - (\boldsymbol{\eta}^* - \boldsymbol{\eta}^{[i]})' (c_{\boldsymbol{\eta}}^2 \boldsymbol{\Sigma})^{-1} (\boldsymbol{\eta}^* - \boldsymbol{\eta}^{[i]}) \right)$$

That is equivalent to generate a random variable, say  $\boldsymbol{\delta}$  from  $\mathbf{N}(\mathbf{0}, c_{\boldsymbol{\eta}}^2 \boldsymbol{\Sigma})$  and add it to  $\boldsymbol{\eta}^{[i]}$  to

propose  $\boldsymbol{\eta}^* = \boldsymbol{\eta}^{[i]} + \boldsymbol{\delta}$ , where  $\boldsymbol{\Sigma} = \begin{bmatrix} c_v^2 & r \sqrt{c_v^2 c_s^2} \\ r \sqrt{c_v^2 c_s^2} & c_s^2 \end{bmatrix}$ ,  $c_v^2$  and  $c_s^2$  were determined after tuning at

the end of Stage 1 and the correlation  $r$  between samples of  $\log(\nu)$  and  $\log(s^2)$  is based on samples drawn during Stage 2.

To determine the acceptance ratio  $\alpha = \frac{p(\boldsymbol{\eta}^* | ELSE)}{p(\boldsymbol{\eta}^{[i]} | ELSE)}$ , we evaluated this ratio in a numerically

stable manner as:

$$\alpha = \exp \left( \log p(\boldsymbol{\eta}^* | ELSE) - \log p(\boldsymbol{\eta}^{[i]} | ELSE) \right)$$

To implement this Metropolis sampling strategy, we first generated  $u$  from a Uniform(0,1) distribution. If  $\alpha > 1$  or  $u < \alpha$  then set  $\boldsymbol{\eta}^{[i+1]} = \boldsymbol{\eta}^*$ . Otherwise, if  $u > \alpha$ , then set  $\boldsymbol{\eta}^{[i+1]} = \boldsymbol{\eta}^{[i]}$ . The Muller [1] tuning procedure was adapted as follows to determine  $c_{\boldsymbol{\eta}}^2$ .

- 1) For the last 10 cycles, the rate of acceptance is greater than 60%, increase  $c_{\boldsymbol{\eta}}^2$  by a factor of 1.2.
- 2) For the last 10 cycles, the rate of acceptance is less than 10%, decrease  $c_{\boldsymbol{\eta}}^2$  by a factor of 0.7.
- 3) After the burn-in, keep  $c_{\boldsymbol{\eta}}^2$  constant and monitor subsequent acceptance rates to ensure that they fall within 25 to 75%.

Stage 4: Jointly sample  $\log(\nu)$  and  $\log(s^2)$  using a bivariate Gaussian proposal density with fixing value of  $c_{\boldsymbol{\eta}}^2$  at the end of Stage 3. After burn-in, save all samples on  $\nu$  and  $s^2$  using MH with the bivariate Gaussian proposal density.

## REFERENCES

- [1] Muller P., A generic approach to posterior integration and Gibbs sampling. Technical Report 91-09 [on line] (1991) [http://www.stat.purdue.edu/research/technical\\_reports/1991-tr.html](http://www.stat.purdue.edu/research/technical_reports/1991-tr.html) [consulted June 10 2014].
- [2] Yi N., Xu S., Bayesian Lasso for quantitative trait loci mapping, Genetics. 179 (2008) 1045 - 1055.